HOW DID THE PRODIGAL SON SAVE HIS KIN

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Abstract. Some aspects of the interaction between science, math contests and advanced mathematical education are considered.

Keywords: math contests, advanced education, educational resources.

Introduction

The content of elementary mathematics is changing constantly; in general it becomes broader all the time. For example, nobody will deny that the algorithm for dividing natural numbers is a part of elementary mathematics; nevertheless, in the middle ages people who were able to use it were awarded scientific degrees.

It is a tradition that the words "elementary mathematics" are connected with school only. This is not quite correct. Of course, no definition in the mathematical sense is possible. Trying to list the parts of elementary mathematics, we include Euclidean planimetry and stereometry, linear operations with plane and space vectors, scalar, pseudoscalar and vectorial products, the greater part of combinatorial geometry, elementary number theory, equations and systems solvable in radicals, algebraic inequalities, elementary functions and their properties, the simplest properties of sequences and the combinatorics of finite sets. There are many mathematicians however who include also elements of graph theory, the simplest combinatorial algorithms, the simplest functional equations in integers, etc. There are parts of mathematics which definitely should not be included; we can mention the methods which are effectively used only by a small amount of mathematicians as well as methods which, though used widely, demand a specific and advanced mathematical formalism.

We can give the following approximate description of elementary mathematics. Elementary mathematics consists of: 1) the methods of reasoning recognized by a broad mathematical community as natural, not depending on any specific branch of mathematics and widely used in different parts of mathematics, 2) the problems that can be solved by means of such methods.

Math competitions and elementary mathematics

The greater part of "responsibility" for the rapid growth of "elementary mathematics" during past decades lies in mathematical competitions. Examining problem sets of

olympiads, contests and preparatory sessions shows that a number of ideas and methods elaborated in advanced areas of mathematics have burst into this field.
There are deep reasons for such a situation. The traditional problem areas become exhausted; we can see it, e.g., from the fact that repetitions of ideas and even problems become more and more frequent even on high-level olympiads. So the organizers and problem composers are in a constant search for new areas and ideas.
One of the characteristic and most important examples is broad uses of the so-called "general combinatorial methods", see, e.g., [1].

In traditional school mathematics, the normal way of thinking is "from the properties of elements towards the properties of the whole set". The same holds true also in most of our everyday life. Nevertheless, in mathematics we can find a number of converse examples, when the analysis goes from the whole set towards individual elements of it. The most elaborated methods of the first kind in "olympiad mathematics" are mathematical induction and invariants, the most useful methods of second kind are the method of extremal elements and the mean value method. Together with "technical disciplines" such as equations and inequalities, geometrical calculations, enumerative methods of combinatorics, incidence theorems, calculus and algorithmics these four methods are the main part of any high-level olympiad preparation program. The examples of their uses are very often taken from the "great science".

The changing role of math Olympiads

As we can learn from many written and oral sources the number of lessons devoted to exact disciplines in school is decreasing in many countries, especially in post-"socialist" ones. Of course, in some sense this can be compensated by introducing new technologies into education. Nevertheless, today not all teachers are ready to explore these advances. So the official curriculum in mathematics today is far from the level of 1980s. Of course, advanced topics were the first to go, and advanced mathematical education in schools met the danger of becoming occasional and disintegrated.

In this situation mathematical olympiads appeared to be a very strong consolidating factor. The Olympiad "curriculum" wasn't changed; it was developed further in an essential way. The standards that were elaborated in the olympiad movement during many years in some sense became the unofficial standards for advanced education in mathematics. There are a lot of topics that are not included in any official school program but nevertheless are discussed regularly with all students interested in mathematics.

The other positive feature of mathematical contests is their stability. Teachers are aware that the olympiads will be held, and they can organize their activities and encourage their students to work additionally for a clear and inspiring aim.
So math competitions, which had often been characterized as "elitist", "discouraging", "far-from-life" etc., appeared to be the strongest support to advanced education at schools in many countries, see, e.g., [2] and [3].
All this sets new tasks for mathematical olympiads. The competitive factor is still extremely important, but also the educational factor of them has become very significant.

From the previous discussion it is clear that math contests today cover a broad spectrum of mathematics. It is particularly important also because olympiad and contest problems from previous years are broadly used afterwards in everyday teaching practice.

The main criteria accordingly to which the set should be well – balanced are:

- it should cover main areas of school mathematics: algebra, geometry, number theory and combinatorics;
- it should contain both problems of a deductive nature and problems of an algorithmic nature;
- "discrete" mathematics and "continuous" mathematics both are to be represented.

It is clear that all these desires hardly can be realized in a small set of five or so problems. Indeed, there are very few sets of problems which can be called satisfactory from all the points of view mentioned. Good balance can and must be achieved during the whole school year.

As contest problem sets usually contain "easy", "medium" and "hard" problems, they provide resources for teachers' activities with all levels of students. Some successful textbooks and a lot of problem books strongly based on contests' content have been developed and published. They have become regular tools for many teachers. In this way contests have also broadened the curricula of math education on an advanced level with those topics brought into them from the "great science" (see above).

There is a constant need for new (at least formally) problems to be used in math competitions. Of course, it should be very nice if we always could sit down and compose an original problem set accordingly to the general demands developed for it. Unfortunately, inspiration traces its own paths and often does not pay attention to our call. In the situation that each year there should be proposed, e.g., ≈250 problems for competitions of all-Latvia scale only, some "industrial" tools must be developed. Some of such tools are described in [4]. They can be used also in developing teaching materials for regular lessons.

An approach based on analysis of contest problems all over the world was developed within **La**tvian-**I**celandic **Ma**thematical project **LAIMA** ("laima" means "happiness" in Latvian). In the opinion of its organizers, there are relatively few basic ideas and relatively few basic approaches which cover almost all of what the international mathematical community has recognized as worthwhile to be included regularly in the search for and promotion of young talents. A very detailed classification of competition problems was developed (its full description consists of approximately 50 pages), and the problems are composed so that in the long run all the "cells" of classification are filled more or less equally. The books developed on the basis of this classification and the corresponding composed problems provide a balanced system of teaching aids suitable on a large scale. For more information, see [6] and [7].

Conclusions

Math competitions form a dynamic system of activities growing both quantitatively and qualitatively. Due to the active collaboration of teachers, researchers in mathematics and related areas (especially theoretical computer science) and other enthusiasts they are stabilizing and stimulating advanced math education in a number of countries, providing also broad possibilities for cooperation between science and education on both national and international scales.

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