ICME-11 Discussion Group 19

The role of mathematics competitions
and other challenging contexts
in the teaching and learning of mathematics

Part I: Mathematics Competitions

This part is written as a direct response to the four questions posed by the Discussion Group.

A. Can it be said that in mathematical competitions — and in challenging mathemat -ical contexts in general — what is commonly known as elementary mathematics both continues to thrive and inspires young minds?

We should find a better name than elementary mathematics. The public tends to confuse it with elementary school mathematics. Perhaps **intuitive mathematics** is a better alternative. Although intuition plays a role in all creative mathematics, we are particularly interested in problems that can be solved with the right intuition, *without* a lot of heavy machinary.

One of the ambitions of the *late* Peter O'Halloran is to make intuitive mathematics, to use my term, a respected area of research in mathematics, with established graduate programs and courses. While this is still a hard-sell in very traditional universities, others are probably ready for it provided that enough work is done to make it work.

A built-in problem with mathematics competitions is what motivates people to enter them. Ideally, they should do so for the thrill of the challenge in tackling hard problems. More often, the principal aim becomes trophy hunting, competition for the sake of competition. This is a serious issue. If the competition is too hard, nobody will come. If it is too easy, there is no point in coming. It is a delicate balance to have mathematics competitons continue to thrive and inspire young minds.

B. To what extent do problems set in mathematical competitions adequately reflect the variety and richness of mathematical activities in problem solving?

Taking all mathematics competitions in totality, they certainly cover a great diversity of topics. Individual mathematics competitions tend to have more limited scope, even the International Mathematics Competition. For instance, problems are often classified as I.M.O. by level, but equally often, they have been classified as I.M.O. by type. In some sense, this is necessary, as each mathematics competition has its own tradition. Thus it is important for students to have experience in more than just one. On the other hand, many students are probably taking part in too many mathematics competitions for their own good. They need time to settle down and reflect on what they have learned from their experience.

The International Mathematics Tournament of the Towns is arguably the best mathematics competition in terms of the variety and beauty of its problems. There is always an elegant idea behind each of them.

C. Are there mathematics and mathematical problems apt for challenging all students?

Intuitive mathematics and its problems are apt for challenging all students. The absence of heavy machinary means that the problems are easy to state and easy to understand. This is an absolutely necessary condition for an appealing problem. It may be desirable to state it in everyday terms, even though the so-called practical situation is often contrived, or even hilariously ridiculous.

D. How can competitions and other challenging mathematical contexts motivate and foster mathematical creativity with students at large?

What we need are follow-up activities. Mathematics competitions are good places to introduce topics not often encountered during school work. Once exposed to these new ideas, the students are often intrigued sufficiently to pursue the topics further on their own. Some structured guidance could be immensely beneficial, leading the students to mini-research projects.

Part II: Other Mathematical Challenges

This part is based on the experience in Edmonton, Canada.

A. SNAP Math Fairs

People often think of the SNAP Math Fair as a variant of the traditional Science Fair, but the two events are fundamentally different. This is implicit in the acronym itself. S stands for student-oriented. N stands for non-competitive. A stands for all-inclusive. P stands for problem-based. Usually, the Science Fair is a static display, by elite students for adults, of their research projects. The SNAP Math Fair is a dynamic carnival, where all the children constitute both the exhibitors and the audience, around booths presenting short interactive puzzles.

The inauguaral SNAP Math Fair in Edmonton was held in a shopping mall in the fall of 1997. The entire student body of a small elementary school took part. They had a rehearsal in the school two days earlier, when the children had a chance to visit the booths of one another. In the actual SNAP Math Fair, the audience were primarily parents and teachers, as well as some shoppers who got more than they bargained for.

A SNAP Math Fair needs practically no expenses to run. Most schools which do them spend two to three weeks preparing for it. Our target group are students in grades four to six. In the setting of our elementary schools where a home room teacher is responsible for most of the subjects, it is easy to find time in the schedule to work on the SNAP Math Fair, and many teachers make it an interdisciplinary project, incorporating industrial arts, creative writing and even computer graphics. In the junior high schools, it is mostly the mathematics classes that are being used. While this seems to take away a disproportionately large amount of time needed to cover the curriculum, teachers have found that they can make up for lost time very quickly, because of the heightened interest and attention of the students after the SNAP Math Fair.

To run a SNAP Math Fair, a teacher first assembles a collection of interactive puzzles. These are distributed without solutions, and the students, usually in groups of two or three, will work them out before figuring out how they will present it. This way, they have a firm understanding of what is involved, and are in good position to offer hints to the audience when they are stumped. It involves them in a teaching situation. Apart from problem solving skills, the students gain proficiency in oral communication and confidence in themselves. In short, they gain poise.

The SNAP Mathematics Foundation was founded in 2004 primarily to promote the SNAP Math Fair. It provides in-service training for teachers who are interested in doing SNAP Math Fairs, and support to schools which are doing them. The most urgent request from the teachers are suitable problems for their students. The Foundation has a website (http://www.mathfair.com) and publishes the Math Fair Booklet (available in English, French and Spanish) by Ted Lewis.

Prior to that, the University of Alberta has incorporated the SNAP Math Fair into their mathematics course for students in Elementary Education. This course is offered by the Faculty of Science

on contents, as opposed to curriculum and intruction courses offered by the Faculty of Education on methodology. Part of the course requirement is to do a SNAP Math Fair. We used to take the class on the road to a particular school, but the program has been so successful, and the waiting list growing so impossibly long, that now the schools come to the university for it. Since there are three such classes in each academic year, the SNAP Math Fair takes place three times a year, bringing about 500 school children to our campus on each day. It is a most successful outreach program. Many schools who have come will follow up with their own SNAP Math Fair.

B. Math Unfairs

On the day of an on-campus Math Fair, about 250 students will arrive by school buses at 9:30 a.m.. The SNAP Math Fair area can hold about half that number. So the other half of the students are taken to classrooms. We have mobilized our graduate students, in both the Mathematics and the Computing Department, to organize this complementary event called the Math Unfair. Mostly, they teach the children mathematical games. In recent years, they have also presented pencil-and-paper puzzles which are unsuitable for the SNAP Math Fair. The graduate students will circulate among the children and provide hints and assistance. The accompanying teachers often work on the puzzles, either by themselves or along with their own students. At 10:30 a.m., the two halves trade places. The same routine is observed in the afternoon session which runs from 12:30 p.m. to 2:30 p.m..

We plan to follow up on this by publishing a puzzle magazine aimed at students in grades 4 to 6. Students will be invited to submit solutions.

C. Math Camps

Since 2000 and every other year, the Canadian Mathematical Society has been sponsoring a Mathematics Regional Summer Camp in Edmonton. Initially, it lasted seven days, and now it runs to ten. Students live in a residence on campus during the whole period. Except for an excursion day in the middle of the camp, there are daily classes organized into mini-courses or topics sessions, and many varied mathematical experiences. Students are also given every opportunity to work with one another, from solving problems to conducting mini-projects. Many mathematically gifted students tend to be loners. Past camps have opened up the horizons for many of them.

Participation is by invitation only, largely based on performances in various mathematics competitions. This is the best reward one can give to the top performers.

D. Math Clubs

The most sustaining event is the Saturday Mathematical Activities, Recreations and Tutorials, a program which has been running continuously since 1981. It meets from 2:00 p.m. to 3:00 p.m. every Saturday in the months of October, November, February and March. There is no fixed agenda. Most of the time, the students, usually numbering just under twenty, will work together on mathematical problems. Now and then, there may be a special lecture on a specific topic of interest to them. It should be emphasized that this is not a class, but a club, where children come to meet others with similar ability level and similar academic interest. Over the years, more than thirty research papers by club members have been published, written before they have graduated from high school. A recent project leads to a commercial product on top of a publication.