HOW TO MAKE THE FAMILY HAPPY

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Introduction

The question mentioned in the title is as old as the mankind itself, and almost everybody has tried to solve it in his own way. The "mathematics education" is also a kind of family, and it also can't be happy if both members aren't equal in their rights and duties. Unfortunately, this is hard to achieve.

There are 4 types of educators:

	In mathematics	In didactics
A	Strong	Strong
B	Strong	Weak
C	Weak	Strong
D	Weak	Weak

In our opinion, those of type C are the most dangerous: they can convince their students of absolutely wrong things so effectively that the situation becomes totally unimprovable. So the role of mathematics should in no way be underestimated while preparing the educators; unfortunately, this often happens in traditional study programs (for bachelors, masters, doctors). Moreover, at our opinion mathematics is such a well – organized discipline, that the didactics of it is almost predetermined by the inner logic of the subject; so recognizing this logic in each separate case is crucial for successful teaching.

As for teaching/learning methods, we should remember that people themselves are learning almost exceptionally from examples, and this is how the teaching/ learning process usually is organized. However, it has been proved in the form of exact theorems in theoretical computer science (see, e.g., [1]) that the possibilities of obtaining general rules from series of experiments are very limited. So other models of inference must be developed and used.

All this creates the necessity of research and study programs for educators in mathematics with strong theoretical components both in mathematics and computer science. On of possible attempts has been made in University of Latvia, where a doctoral program "Modern elementary mathematics and didactics of mathematics" has been established.

The concept of modern elementary mathematics

It is a tradition that the words "elementary mathematics" are connected with school only. It's not quite correct. Of course, no definition in the mathematical sense is possible. Trying to list the parts of elementary mathematics we include Euclidean planimetry and stereometry, linear operations with plane and space vectors, scalar, pseudoscalar and vectorial products, the greatest part of combinatorial geometry, elementary number theory, equations and systems solvable in radicals, algebraic inequalities, elementary functions and their properties, the simplest properties of sequences and the combinatorics of finite sets. There are many mathematicians, however, who include also elements of graph theory, simplest combinatorial

algorithms, simplest functional equations in integers, etc. There are parts of mathematics which definitely should not be included: we can mention the methods which are effectively used only by a small amount of mathematicians as well as methods which, though used widely, demand a specific and advanced mathematical formalism.

We can give the following approximate description of elementary mathematics. Elementary mathematics consists of: 1) the methods of reasoning recognized by a broad mathematical community as natural, not depending on any specific branch of mathematics and widely used in different parts of it, 2) the problems that can be solved by means of such methods.

Evidently, such a concept of elementary mathematics is historically conditioned.

Many new areas of mathematics, especially in the discrete and algorithmic parts of it, are still today exploring elementary methods as the **main tool**. Obviously it can be explained at least partially with the fact that the natural questions there have not yet been exhausted, and natural approaches are therefore effective.

The movement of mathematical contests, especially of mathematical olympiads, has made an important service to elementary mathematics. Becoming a mass activity, the system of math competitions created a large and constant demand for original problems on various levels of difficulty. Clearly school curricula couldn't settle the situation, and the organizers of the competitions turned to their own research fields where they found rich and still unexhausted possibilities.

One of important results that originated from the "olympiad mathematics" was the identification of the so called general combinatorial methods (mean value method, invariant method, extremal element method, interpretation method) (see, e.g., [2]).

Elementary mathematics was first officially recognized as an independent branch of mathematics in 1995 when the Latvia's Council of Science published the formal structure of science in Latvia, "Modern elementary mathematics and didactics of mathematics" (further MEM/DM) being one of 12 parts of mathematics. Since then, master and doctoral degrees are awarded in this area.

We stress especially that with this decision the didactics of mathematics is also recognized as a **part of mathematics**.

The formal structure of the doctoral program

From the formal point of view there is only one doctoral program in mathematics in the University of Latvia (the leading scientific/ educational institution in the country with high international reputation); it has 8 branches (algebra; differential equations; geometry and topology; …; MEM/DM). So doctoral students must fulfill the general requirements for future doctors in mathematics. At first, it means the examination in general mathematics; each student has to select (together with his supervisor) 20 questions out of the list of 92 questions. We give 10 characteristic examples of questions usually recommended for doctoral students in MEM/DM:

- Most important axiom systems of set theory: Zermelo-Fraenkel and Goedel-Bernays systems. Corollaries from the axioms. The axiom of choice and its equivalents: Zorn's lemma, Kuratowski's principle etc.
- Elements of algebraic and analytical number theory. Field of algebraic numbers and its basic properties. Basics of ideal theory. Classical transcendence proofs. Central results on prime distribution. Riemann ζ-

function and Dirichlet L-function. Fast algorithms for factorization and for primality.

- Combinatorial structures. Elements of enumerative combinatorics. Transversals, latin squares, block-schemas, finite geometries. Generating functions and their algebra. The method of recurrence relations. Moebius inversion function. The orbit method.
- Ramsey theory. Classical Ramsey numbers and their generalizations. Ramsey-type structural theorems in number theory. Ramsey type results in geometry, algebra, mathematical analysis, combinatorics. Classical minimax theorems.
- Classical fast algorithms: Sorting algorithms. Algorithms for arithmetical operations. Algorithms for computing polynomials. Algorithms for operations with matrices.
- Main formal concepts of algorithm and their basic properties. Turing machines, normal algorithms, recursive functions and their equivalence. Reducibility and its formalizations. Algorithmically unsolvable problems, Kleene-Mostowski hierarchy. The characterization of recursively enumerable sets through Diophantine predicates.
- The concept of probabilistic algorithm. Probabilistic Turing machines, their principal possibilities in set recognition compared with those of deterministic Turing machines. Advantages of probabilistic Turing machines and various types of automata over their deterministic counterparts from the complexity point of view.
- Elements of dimension theory. Small inductive (Menger-Urison) dimension *ind*. Large inductive (Brauer-Čech) dimension *Ind*. Layer (Lebesque) dimension *dim*. Dimension of the subsets of Euclidean space. Basic properties of the dimensions of separable metric spaces. Inequalities between various dimensions. Zero-dimensional spaces.
- Main methods of proving algebraic inequalities. Classical inequalities and their generalizations. Isoperimetric problem and its variations. Fast algorithms in the analysis of systems of inequalities.
- General combinatorial methods and their applications. A concept of invariant method, mean value method, extremal value method, interpretation method. Formal deductive systems. Bases of the systems of functions in the algebra of logic. Impossibility proofs in automata theory. Lower complexity bounds for combinatorial algorithms. Elements of the complexity theory for computations.

As we see there is a strong stress on the discrete/ algorithmic side of mathematics. This is explained by the growing role of it in science and applications, which must be reflected also in the education on all levels.

Doctoral students must pass also 1) the examination in their “narrow speciality” that requires studies in didactics, 2) the examination in the “second” foreign language.

Since 2006 the formal requirements for receiving the doctoral degree in Latvia include only 2 reports on international conferences and a quite smooth demand that the “main results” must be published in internationally reviewed journals/ proceedings/ paper collections etc. Nevertheless, for doctoral students in MEM/DM the unofficial standard is not less than 5 conferences and 10 publications of the above type. Also at least one published teaching aid based on the performed research is considered as very welcome.

The main directions of the research

During the last 30 years – both in Soviet period and after regaining the political independence – the education in Latvia is reformed almost without interruption. Generally speaking, it has become more "colorful" but also, unfortunately, more sketchy. The great aim to make the education more **human** than it was in the Soviet period was substituted by the aim to make it more **humanitarian**. So significant harm was done to the teaching of exact disciplines at middle and high school, and serious efforts must be applied now to turn the wheel back. The main task of researchers in the area of mathematical education in Latvia is to find the ways how to do it as fast as possible. It seems that there is no hope to return to the previous number of lessons for mathematics (at least at this moment); so something principally new must be found.

Three main directions that are chosen for the research are as follows:

1) the integration of teaching various topics on the basis of unifying ideas discovered in the area of modern elementary mathematics,
2) the possibilities provided by ICT and deeper understanding of the nature and role of algorithmics in mathematics,
3) the development of the system of out-of-class activities, especially mathematics contests.

Although each possible doctoral student is offered to select a topic from the abovementioned areas, he has a full right and is even urged to come up with his own proposals.

Some results

There are some findings made by doctoral students and confirmed by the praxis that have found serious applications in the educational system of Latvia:

1) the classification of the contest problems and solution methods within Latvian-Icelandic project LAIMA (see, e.g., [3] and [4]),

2) the classification of Internet resources in school mathematics and creating a structured survey of them within the Latvian Education Informatization System project during 1997-2005 (see, e.g., [5]),

3) the investigation of the possibilities to use the method of invariants as the unifying factor in teaching some hard topics (see, e.g., [6]),

4) the investigation of the possibilities to use the method of interpretations as the unifying factor in teaching seemingly different topics (see, e.g., [7]),

5) the investigation of the principal questions about problem setting for math competitions (see, e.g., [8]),

6) the investigation of the possibilities to integrate elements of combinatorics into the curricula for middle grades (see, e.g., [9]).

Conclusions

The close integration of doctoral studies in didactics of mathematics with modern elementary mathematics has made a good service for both and has led to real improvements in the education on middle and high school levels.

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