The key component?

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Abstract
Are there really three main components of teacher education – mathematical, educational, and didactical – and should we be aiming for an "appropriate balance" between them? Are there perhaps missing components? Or is one of the named components "more equal" than the others? We outline a new CPD venture based on the clear view that mathematics teaching has to be rooted first and foremost in elementary mathematics.

THE SETTING

Modern schooling obliges teachers to observe all manner of bureaucratic requirements. But these are to a large extent subject-independent and can be handled relatively routinely; they should certainly not be allowed to distort the spirit of on-going professional development for *mathematics* teachers.

More importantly, any attempt to teach mathematics effectively obliges us to consider broader educational issues, and to understand the associated didactical judgements and choices which the teacher makes – by active choice or by neglect – at every turn.

However, the whole thrust of liberal education recognises that learning takes place in the context of *specific disciplines* – disciplines which have proved their value over many centuries (sometimes millennia), and which fit naturally at a young age. We do not begin with one monolithic "educational dogma", which is then interpreted in the context of each individual subject; rather, our shared educational goals are an amalgam of ideas and experiences from what different subjects and activities contribute to the development of young minds and bodies. Such shared goals sometimes reflect what is *common* across several different areas – as in the very notion of "discipline", both in the sense of "personal application and persistence", and of a "coherent" heritage of facts, techniques, and currently accepted analyses. But as soon as we go beyond this vague commonality *between* subjects, the details begin to diverge: "analysis" and "reasoning" have very different meanings in history, in science, and in mathematics, as does the notion of "currently accepted". Some disciplines engage with the complexities of society as it is, whilst others operate in an abstract mental universe. Some disciplines encourage the expression of personal opinion, whilst others struggle to develop "objective truth". Some view all human experience as their oyster, whilst others restrict to what can be counted or measured. Hence the didactical judgements and choices we make are also likely to be subject-specific: there is no central theory of "didactics" – only didactics *of some specific subject*.

So we conclude that, whilst the educational and didactical components play an important role in the education of mathematics teachers, they depend on a clear feeling for elementary mathematics itself. Hence *the mathematical component holds the key to the professional development of mathematics teachers.* This is not to say that "the best mathematicians make the best mathematics teachers"; rather, that given any individual teacher (with their

personality, their experience, their attitudes to children, their natural style, etc.), the most effective way to improve their appreciation of the different components of mathematics teaching, to sharpen the didactical judgements they make every day, and to optimise their overall effectiveness, is to strengthen their understanding of, their love for, and their appreciation of the discipline they profess to teach.

THE NATIONAL MATHEMATICS TEACHERS' SUMMER SCHOOL

Until relatively recently, the mathematical component in teacher education has often been "de-emphasised"; but there are now signs of a welcome recognition of its importance. Nevertheless, it would be wrong to suggest that the view outlined in the opening section (namely of the *central* importance of the mathematical component) is widely shared in mathematics education circles; so in the current climate such a view is likely to struggle to make much of an impact.

This was the setting in which the first *National Mathematics Teachers' Summer School* was launched in the UK in 2007. Its aim was to immerse good teachers in a mathematical world for a highly intensive week, not only to open their eyes to elementary mathematics in a new way, but to use this experience as a way of encouraging reflection upon the didactical and educational components. It was inspired by the *National Mathematics Summer School* for secondary students (aged 14-16), started in 1994 – though we have since become aware of similar teachers' summer schools elsewhere (like that associated with the *Park City Mathematics Insitute* http://pcmi.ias.edu/current/program_highschool.php).

The clientele

The Summer School was intended to provide a life-changing experience for promising secondary mathematics teachers with 2-5 years experience, to provide a meeting-place where they might make contacts that last, and where they might begin to develop a view of elementary mathematics that then continues to evolve throughout their teaching careers.

The venue

The Summer School was held in a modern Cambridge college, which provided a physically closed "family atmosphere", in a delightful setting, with all the necessary facilities on site – yet within 5-10 minutes walk of a world with rich mathematical associations.

The daily programme

The city of Cambridge is remarkably compact, with hidden bits of mathematical history round every corner. On the opening day, after registering at 2pm, participants were grouped into teams of four and sent off with a map and clear directions to find answers to 20 quiz questions in the form of a *Mathematics trail* round the city. This was designed partly as an ice-breaker, partly to introduce them to the city, and partly to emphasize what can happen when one views everything through a "mathematical lens". Most delegates had very little sense that mathematics has a history, or that the contributions of mathematicians are celebrated all around us: for example, three of the four roads round the college are named after Sylvester, Adams and Herschel, and the first question challenged them to find out what these individuals

are celebrated for. The trail led them past the *Isaac Newton Institute* – where they were challenged to identify the sculptor of the intriguing shapes outside; through Trinity College – where they had to list all the mathematicians feted in the college chapel; and into Caius College – where they had to identify the mathematical themes in, and the mathematician maker of, the stained glass windows which decorate the dining hall.

After supper that same evening, this hands-on introduction to the local history of mathematics was followed up with a lecture on "800 years of Cambridge mathematics". Even the organisers were surprised later in the week at an unexpectedly powerful response to this "first hand" contact with the world of mathematics (past and present). Around half the participants signed up to be taken round the Newton Institute and the Centre for Mathematical Sciences (rather than go punting), and many were visibly moved to be in physical proximity to those (such as Stephen Hawking) who live each day in the remarkable mental universe of mathematics.

In general, the evenings (after 5pm) were reserved to convey aspects of the "lighter" side of mathematics. But on the first day 4.30-6 was used for the first session of four extended sessions devoted to a mini-course on Euclidean geometry. Most participants had no conception of the nature of a logical hierarchy, and had never worked through, nor reflected upon a semi-formal treatment of elementary geometry. They did not find it easy – and there were at first signs of a minor rebellion from those in the back row. But this proved to be a necessary learning experience and almost all delegates emerged the stronger.

> "It was an incredible opportunity for us all, and I do think it will have a huge impact on my teaching. If nothing else, I hope I'm passing on the tremendous enthusiasm I have for maths again."

Each day began at 8.45 with a 90 minute session in groups of 10 tackling a set of problems on a given theme. This proved to be an accidental masterstroke. Each group was led by one of the central team or by an experienced teacher, with very little guidance beyond the basic structure (each group was encouraged to write up and post solutions each day on a large noticeboard) and explicit encouragement to look after any who might find the experience daunting. In the event each group developed its own character – like a family – and served as an admirable antidote to the front-led sessions which formed the bulk of each day's programme.

Morning coffee and afternoon tea were provided out in the summer sun, on a walkway adjoining the daily "exhibition" – where each day a new agency or organisation displayed material the teachers might not otherwise be aware of.

The other three 90 minute slots each day (up till 5pm) were allocated to mini-courses of 3 sessions – with one session each day (on elementary algebra: errors and misconceptions; fractions, ratio and proportion; an approach to extension work in mathematics).

From 5pm each evening the mood changed, but the intensity continued. Sessions became shorter (1 hour) and lighter (a mathematical pub quiz, the mathematics of juggling; forensic statistics, beauty in mathematical puzzles, Why does a tripod have three legs?, mathematics and ancient Greek philosophy), but always designed to open up new vistas in the teachers' mathematical universe.

The response

The initial response from delegates was overwhelming.

> "Thanks for a brilliant week."
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> "Although I found it extremely hard work, I really enjoyed it too and am very grateful for the experience."
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> "it really made me think, not just about my teaching, but also about my own philosophy of mathematics. Thank you for making me think again."
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> "I think it had such an impact because of the length [1 week may seem short to those in other countries; but most CPD in the UK is restricted to a single day, 10am-3.30pm]: courses that only last a day can only cram in so much – and as with pupils, when you try to teach too much in too short a time, not much gets absorbed. Also the variety of activities (group work, Euclidean geometry, applications to teaching, outside speakers, etc.). Being the week before we returned to school was great: I returned energised and ready to try new things with the fresh start that a new academic year brings."
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> "I do hope you are able to make this an annual event and that in the long term it has an impact on the teaching of mathematics in this country."

We now face the challenge of replicating this initial success – and it is not proving easy. Because of difficulties in recruiting for the first year, we decided (for 2008 only) to try to influence more experienced mathematics teachers, so that they might then encourage younger teachers in their own charge to attend in subsequent years. Noone warned us that recruiting more experienced teachers would be infinitely harder than recruiting enthusiastic raw recruits! Nevertheless we shall stick to this plan for 2008 before reverting to the original target group from 2009 onwards.

Experience this summer may force us to modify our analysis. But our tentative conclusions thus far remain consistent with the observation that many ambitious schemes in recent decades have failed because they ignored key features of elementary mathematics; and those teachers and schools who have apparently prospered despite these failures of public policy have done so because their teachers and traditions continued to respect the mathematical component as *primus inter pares*. Hence, it may be that, whilst educational and didactical considerations are important, they only make sense when viewed through a strong mathematical lens. If this is indeed the case, then *it is the mathematical component that has priority and that holds the key to the professional development of mathematics teachers.*