Chinese Elementary Teachers' Understanding and Representation of Mathematics Concepts in Classroom: Relationship between Pedagogical Content Knowledge, Instructional Practice, and Mandated Curriculum Materials

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Abstract

The establishment of mathematics curriculum standards is an important US policy initiative to urge teachers improve mathematics understanding and representation in their classrooms. The assumption is that the well-designed mandated curriculum helps improve teachers' knowledge of mathematics and its representation and thus, the quality of their instruction. Although a careful examination of this assumption is necessary, it is difficult to explore this assumption in US school contexts where the mandated curriculum standards and materials are either absent or emergent. Although China provides an alternative context for this examination, few existing studies explore directly how the national curriculum standards and materials are related to the mathematics knowledge that Chinese mathematics teachers developed and to their representations of this understanding to their students in their classrooms. Drawing surveys, observations, and relevant curriculum materials from 7 fourth to sixth grade Chinese mathematics teachers working in three schools in the same district, this Spencer Foundation Sponsored study explores the relationship between teachers' understanding and representation of mathematics in their classrooms and the mandated curriculum materials that they are required to use in instruction. With this examination as a base, it critiques several theoretical assumptions about the US mathematics education reform.

A. Purposes of the proposal

Effective mathematics instruction relies on teachers' deeper understanding of what they teach (Lampert, 1990) and their flexible representations of this understanding to students (Shulman, 1987). The establishment of mathematics curriculum standards is an important US policy initiative to urge teachers improve mathematics understanding and representation in their classrooms (National Board for Professional Teaching Standards, 2002; National Council of Teachers of Mathematics, 1991). Underlying this effort is the assumption that a well-designed curriculum helps improve teachers' knowledge of mathematics and its representation and thus, the quality of their instruction.

Although a careful examination of this assumption is necessary, it is difficult to do it in US school contexts where the mandated curriculum standards and curriculum materials are either absent or emergent (Hiebert, 1999). However, China provides an alternative context for this examination. First, Chinese students outperformed US peers in mathematics as measured by curriculum-based examination (Chen *et al.*, 1996; Stevenson *et al.*, 1990), US standardized assessments (Geary *et al.*, 1993; Geary *et al.*, 1999), and researcher self-design assessments

(Brenner *et al.*, 1999; Miura *et al.*, 1994). Second, Chinese teachers' instruction follows the nationally mandated curriculum and is checked regularly by curriculum-based examinations at the district level (Wang, 2001). Third, Chinese mathematics teachers developed a deeper understanding of what they teach and flexible representation of this understanding than their US counterparts based on the interview data (Ma, 1999). Fourth, Chinese teachers were also more likely than US counterparts to vary their instructional tasks to hold student attention and more likely to teach students to respond to mathematics problems in a rapid manner (Stevenson & Lee, 1995) and offer increasingly direct and complex explanations (Perry, 2000).

However, few studies explore directly how the national curriculum standards and materials are related to the mathematics knowledge that Chinese mathematics teachers developed and to their actual representations of this understanding in their classrooms. Drawing surveys, observations, and relevant curriculum materials from 7 fourth to sixth grade Chinese mathematics teachers in three schools in the same district, this study explores the relationship between teachers' understanding and representation of mathematics in their classrooms and mandated curriculum materials that they are required to use in teaching. With this examination as a base, it critiques several theoretical assumptions about the US mathematics education reform.

B. Theoretical framework

Two theories are developed to explain the relationship between mandated curriculum, teachers' knowledge, and their instruction. One sees teachers' autonomy in developing their curriculum is crucial for improving their teaching. It contends that a mandated curriculum and teacher relationship will present difficulties for teachers to create, sustain a collaborative culture (Hargreaves & Dawe, 1990), weaken teachers' confidence and ability in making teaching decisions, and destroy their creativity in inquiry into instruction (Cochran-Smith, 2001).

Another assumes that lack of mandated curriculum and too much teachers' autonomy can create problems for teachers to improve instruction. Such an environment allows teachers to hide individual struggles in learning to teach and removes the opportunity for teachers to examine and critique teaching based on shared professional standards and purposes (Ball & Cohen, 1999). It may also weaken intellectual, social, and emotional demands for teachers' interdependence and reduce collaboration to story sharing and informational assistance instead of joint-work that requires consensus of thought and uniformity of action (Little, 1990).

Research supporting either theory is insufficient and fragmented. The first assumption often relies on surveys about teachers' negative feelings and complaints from the contexts of changing curriculum (Helsby & McCulloch, 1996; Thiessen, 2000). However, these feelings and complaints can also be seen as a sign of necessary conceptual discrepancies that will lead teachers to rely more on each other in learning to teach (Hargreaves, 1994).

The second assumption finds its support from two lines of research. First, US teachers' autonomy does not help them pay attention to mathematics concepts taught and corresponding pedagogical implications even though they are engaged in observing and critiquing lessons (Grant *et al.*, 1998). Second, Chinese teachers have a deeper understanding about mathematics content, connections, and representations (Ma, 1999). Their mandated curriculum materials also focus on connections of concepts and use of observations, manipulative, and semi-concrete activities to develop students' abstract representations of problems (Zhou & Peverly, 2005). However, these analyses are descriptive and isolated with little attention to the dynamic interplay between the mandated curriculum, teachers' subject understanding and representation, and instructional practice (Wang & Lin, 2005).

This study is designed to understand and verify such interplay and help build a knowledge base upon which policy makers and program developers can make wise decisions in helping develop teachers' mathematics-specific pedagogy and effective instruction.

C. Data collection

The data of this study came from seven Chinese mathematics teachers from three elementary schools in an urban school district. Two teachers taught all fourth, fifth, and sixth grade classes in a school whose students had lower mathematics performances based on the district level curriculum-based examinations. Three participants taught all the fifth and sixth grade level classes in another school whose students only had average mathematics performances. Two teachers taught all the fifth grade level in the third school whose students had higher mathematics performances. We choose these participants and the schools for this study for several considerations:

First, all the participants were teaching one of the two most difficult mathematics topics in the elementary mathematics curriculum: (1) Fraction world problem involving multi-forms and steps of calculation and (2) area problems of various geometric shapes involving multi-step calculation. Second, all participants worked under the same mandated curriculum and assessment requirements and used the same curriculum materials for their lessons. Third, they represented the range of teachers in the school district teaching different levels of students.

The data of this study came form a larger study sponsored by the Spencer Foundation that explores the relationship between the contrived curriculum and teacher organization, teachers' mathematics knowledge and teaching practice, and student performances in China. These data were:

First, lesson observations. Twenty-one 40-minute mathematics lessons on either topics were used for this study that included three lessons from each of the two classes that each participant taught. All the lessons were videotaped, transcribed and then translated from Chinese into English. These data provided information about the participants' teaching practice.

Second, a questionnaire survey was administered to each participant on their understanding and representation of the mathematics concepts at their grade levels and their general attitude toward mathematics, mathematics learning, and teaching. The survey was adopted from the survey developed by the National Center for Research on Teacher Education, Michigan State University (Kennedy, Ball, McDiarmid, 1993). This data set provides the information for each teacher's understanding of either of the two topics and its representations that they need to teach in their lessons. The survey data were translated from Chinese into English for further coding.

Third, relevant parts of national framework, textbooks, and teachers' manuals were also collected to provide information about mandated curriculum materials that the participants used to develop their lessons. These materials were translated from Chinese into English for further coding and analysis.

D. Methods of inquiry

The following kinds of analysis were conducted to analyze each kind of the data collected for this study in order to address the questions of this study. First, for the lesson observation, each lesson transcript was coded as suggested by Strauss & Corbin (1990) to capture its pedagogical moves including lesson sequence, time, instructional approach, and interaction patterns and then each lesson transcript was coded in a similar manner to identify the representations of mathematics concept in the lesson, including the kinds of concepts presented in each lesson, the sequence in which they were represented, and the ways in which these concepts were related with each other.

Then, the patterns of pedagogical moves and concept representation were compared across different lessons taught by each teachers, lessons taught by the teachers in the schools, and then with patterns of the lessons taught by teachers from different schools to for establishing similarities and differences" (Eggan, 1965) (p.336) so that patterns of pedagogical moves and representations the levels of individual teacher, school, and across schools can be captured.

Second, the survey with each teacher were coded and analyzed for the breadth and depth of each participant's understanding of the concepts and theorems that they needed to teach in the above lessons and then their ideas of mathematics, mathematics learning, and teaching were coded and analyzed following the guideline of survey analysis developed by the NCRTE. These results of analysis from each teacher were compared and contrasted with each other across all the participants to capture the similarities and differences in their understanding of mathematics concepts and theorems and the representations of these ideas in their classrooms.

Third, for curriculum analysis, the relevant parts of curriculum materials for the topics of each lesson including the national curriculum standards, mandated textbooks, and teachers' manuals were first coded for the concepts and theorems required for each lesson and then for the specific suggestions for pedagogical moves and concept representations. This analysis is guided by the adapted guidelines of curriculum analysis developed by the curriculum study of the Third International Mathematics and Science Studies.

In the end, the results from the analyses in the above phases are compared with each other in order to capture the relationship between teachers' pedagogical content knowledge, their teaching practice, and the curriculum materials. Then the relationships identified from this analysis were then contrasted with the four kinds of mathematics instruction as Kush and Ball (1986) conceptualized to identify the opportunities for students to learn mathematics in these lessons. These kinds of mathematics instruction included (1) Learner-focused instruction in which a teacher poses questions and situations for students to explore and formalize ideas and solutions and then challenge and facilitate them to validate and prove their ideas among themselves. Such teaching reflects the idea of mathematics learning as active construction, communication, and examination of mathematics ideas envisioned by the US mathematics standards (Cobb, 1995; Cobb, 1996). (2) Conceptual knowledge-focused instruction in which a teacher supports students to develop a conceptual understanding about the logic and relationships of mathematics ideas which is consistent with the idea of mathematics as a discipline with its logic, relationships, and approaches of inquiry underlying the US mathematics education reform (Ball & McDiarmid, 1989; Lampert, 1992). (3) Factual knowledge-focused instruction in which a teacher defines, demonstrates, and explains mathematics rules, skills, and procedures while students listen, answers questions, and practice them. Such teaching resonates the idea of mathematics as a collection of isolated facts, rules, formulas, and skills and mathematics learning as to internalize definite answers in consistence with direct instruction described in the literature (Smith III, 1996). (4) Sequence-focused instruction in which a teacher assigns, monitor, and provide feedback to students' work and prevent disruptions interfering with students' learning with little attention to the organization of mathematics and theory of learning. Such teaching reflects prevailing teaching practice (Romberg, 1992).

E. Results and conclusions

The study had several shared patterns among these participants in spite the variations among their knowledge and instruction. First, all participants in this study developed an deeper understanding about the topics that they were required to teach as those identified by Ma (1999) which suggested that mathematics as various concepts that connected with each other and students' clear understanding and justification of mathematics concepts and their connections were important part of their thinking about mathematics instruction.

Second, their knowledge and thinking of mathematics, its learning, and teaching were reflected in their pedagogical moves and content representations in several ways. While the sequence of their pedagogical moves looked much alike, which started review, new content instruction, and then to various kinds of practices, in each stage, each participant posed questions and situations for students to formalize ideas and solutions and then them to validate and prove their ideas to the class instead of modeling students concepts and solution directly.

Third, all the participants designed a similar sequence of problems to help students to practice what they learned. It started with a problem requiring direct application of new concept followed by problems with increased levels of difficulty that asked for students to use both new and prior concepts, multi-step proof, and alternative solutions.

In the end, the study revealed also that the mandated curriculum materials contributed importantly to concept representation and practice activities developed in the lessons. However, these materials did not limited participants' autonomy in developing their own objectives, example problems, and practice activities beyond the requirements of the mandated curriculum.

F. Educational Importance

The study suggests that teachers under the mandated curriculum are able to develop a deeper understanding of the mathematics concepts and their relationships as well as content representation. Such thinking and knowledge were clearly reflected in their quality instruction. Mandated curriculum materials could influence teachers' choices of teaching content, use of teaching time, pedagogical discourse, and focus their attention to the important objectives, useful representations, and flexible connections between different mathematics ideas. Such finding supports the assumption that teaching practice can be improved when curriculum materials are mandated in a way that carries enough authority to influence teachers' teaching, provides specific standards, suggestions, and resources for their instruction, and shows the consistence between the various kinds of curriculum materials (Cohen & Spillane, 1992).

It also shows that curriculum materials, although having powerful influence, do not necessarily prevent teachers from developing higher expectations, better example problems, and teaching strategies for their lessons. Instead, teachers, although inexperienced, still have the autonomy to further develop these aspects of their teaching.

The study suggests that the question that we need to ask is not whether or not curriculum standards should be used to improve teaching quality of. Instead, we need to ask how curriculum needs to be structured and what kinds of autonomy are necessary for teachers to develop useful professional knowledge and quality teaching. To answer this question, we need more research focusing on teachers' daily teaching practice and how they made their teaching decisions under various contexts of curriculum.

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