The Evaluation of Mathematics Learning in Context [1]

Verónica Díaz &. Alvaro Poblete[2]
Universidad of Los Lagos, Chile

Summary

Education Reforms currently underway in Iberoamerica focus on improving the pedagogic quality of education in general. In the school teaching context, this has been complemented by various efforts to identify how best to create the conditions necessary to enhance educational institutions in such a way as to ensure that progress takes the form of viable, effective and lasting transformations that enrich and reform pedagogic practice. Similarly, systems to evaluate the quality of education have also been developed.

Over the past few years, various countries in America have participated in the evaluations carried out by the Third International Mathematics and Science Study TIMSS and the Programme for International Student Assessment PISA, and the results in mathematics have placed them consistently among the countries with lowest performance rates. In the light of these results, and with the reform process already underway in various countries, several questions arise with respect to the low performance rates in mathematics: is this due to student characteristics?; to teacher competence in the mathematics-didactics field?; to the method of transferral in the classroom? Unfortunately, the centralised evaluation systems in each country are not designed to provide answers to these types of questions; they merely give performance percentages for the disciplines, and translate them into indicators in their own rankings. This situation contrasts with the objectives of study programmes in countries already implementing the reform process, which consider innumerable, specific abilities associated with mathematics, supported by a contemporary teaching curriculum.

The importance of developing problem-solving capacities and mathematical skills, continues to be an issue of interest from the perspective of education reforms and curricular tendencies in America, as well as of international performance studies in mathematics undertaken over the last few years. In particular, this element is fundamental to mathematics teaching at different levels in Chile today, given that the curriculum is justified, to a large extent, on the grounds that skills must be applied and be useful in everyday situations (Díaz; Poblete, 2001).

On this basis, we developed a learning evaluation for students, on the basis of types of problems and types of mathematical competence. Types of problem solving are classified according to their characteristics as: routine and non-routine, and according to their context, as real, realistic, fantastical and purely mathematical (Díaz; Poblete, 1994,) in the areas of algebra and geometry, in line with the subject matter proposed in the Education Reform. As regards types of competence, we focus on i) knowledge and development of mathematical procedures, ii) solving of routine problems and iii) formulation and solution of non-routine problems (Poblete; Díaz, 2003).

Key Words: *problems solving; mathematical competence; secondary school*

---

[1] This article shows the first results of National Project Fondecyt N°1040035.

[2] Doctora.Verónica Díaz mvdiaz@ulagos.cl & Alvaro Poblete,Ph.D. apoblete@ulagos.cl
Lecturers and Researchers specialized in Didactics of Mathematics. Depto. Ciencias Exactas. Universidad de Los Lagos. Osorno. Chile

Introduction

Based on the Chilean results in all the assessment conducted, we propose a learning assessment model through a national research project that is directly related to the international assessment systems (Proyect Fondecyt 1040035). This model is based on the solving of types of problems and types of students' mathematical competences, and on the secondary school curriculum under Educational Reforms in Latinamerica, and more particularly the Chilean one.

This article develops a proposal to evaluate mathematics performance concentrating on knowledge and skills, based on the resolution of problems in the areas of algebra and geometry.

Education reform in Iberoamerica

There are many problem-solving models ranging from the classic proposals of Schoenfeld (1985) and Polya (1982) that embrace the heuristic concept, to the other, less mathematical, more psychological models of Bransford and Stein (Chamorro, 2003). Problem-solving implies using superior cognitive skills, acquired gradually, that are complexly interconnected and, therefore, difficult to observe or measure.

The theoretical framework of the Third International Mathematics and Science Study TIMSS 2003 explicitly mentions the importance of developing mental dexterity: "solving problems in the framework of mathematical or real life situations where there is very little probability of students having encountered similar items; and the application of mathematical procedures in unfamiliar contexts" (Beaton, 2003). The Programme International for Student Assessment PISA also proposes that mathematical formation requires the ability to apply knowledge, comprehension and mental dexterity in a wide variety of personal, social and work contexts (Gil, 2002).

Over the past few years, various countries in America have participated in the evaluations carried out by the Third International Mathematics and Science Study TIMSS and the Programme for International Student Assessment PISA, and the results in mathematics have placed them consistently among the countries with lowest performance rates. In the light of these results, and with the reform process already underway in various countries, several questions arise with respect to the low performance rates in mathematics: is this due to student characteristics?; to teacher competence in the mathematics-didactics field?; to the method of transferral in the classroom? Unfortunately, the centralised evaluation systems in each country are not designed to provide answers to these types of questions, they merely give performance percentages for the disciplines, and translate them into indicators in their own rankings. This situation contrasts with the objectives of study programmes in countries already implementing the reform process, which consider innumerable, specific abilities associated with mathematics, supported by a contemporary teaching curriculum.

In the specific case of Chile, the organisation of secondary education curriculum openly addresses the need to encourage the development of "the ability to solve problems associated with the skills that enable learners to use tools and procedures based on routine, and the application of principles, general laws, concepts and criteria that help learners to face school situations the same as everyday life situations in a systematic and reflexive

way, having a critical and self-critical disposition" (Ministry of Education, 1998). However, continuous evaluations on a national scale of learning processes in mathematics at primary school level have been characterized by low achievement levels; these results do not reflect either a qualitative improvement in the performance of the teaching staff who are faced with a reformed curriculum, or an effective transferral of the didactic activities to their pupils.

Problems Types and Areas of Mathematical Competences

The study considered the learning evaluation in students, on the basis of types of problems and types of mathematical competence. Types of problem solving are classified according to their characteristics as: routine and non-routine, and according to their context, as real, realistic, fantasy and purely mathematical (Díaz; Poblete, 1994,) in the areas of algebra and geometry, in line with the subject matter proposed in the Education Reform. As regards types of competence, we focus on i) knowledge and development of mathematical procedures, ii) solving of routine problems and iii) formulation and solution of non-routine problems (Poblete; Díaz, 2003).

- A routine problem according to its context, will be classified as *real*, if it is effectively produced in reality and actually involves student activity.

Example: Measure the diameter and length of the circumference in three coins of different sizes with a piece of string. Establish the ratio between the diameter and length of each coin. What can you conclude from these ratios?

- A context problem is *realistic* if it can occur in reality. It is a simulation of reality or a part of reality.

Example: A one year old baby uses, on average, 800 disposable nappies, the equivalent of felling 4 trees to make them. On studying a sample of 125,000 babies throughout one year it was concluded that: 30% do not use disposable nappies, 20% uses 400 disposable nappies and the remainder use exclusively disposable nappies. How many trees would have to be felled in

- A context problem falls into the *fantasy* category if it is the fruit of imagination and does not have any basis in reality.

Example: The race that inhabits the planet Jupiter has only four varieties of hair, four varieties of forehead, four types of nose and four kinds of mouth. How many varieties of face exist in this planet?

- A context problem is *purely mathematical* if it refers exclusively to mathematical objects: numbers, arithmetical relationships and operations, geometric figures, etc.

Example: Find the number that increased by 20 is equivalent to triple the same number.

- Problems are *non-routine* in the sense that the student does not know the answer or any previously established procedure or routine for working out the answer.

Example: Two friends were told a secret at nine o'clock in the morning and warned that they should not tell anyone. A quarter of an hour later, each of them had told three friends, naturally very trustworthy. A quarter of an hour later, each of these friends had told the secret to another three friends, who, in turn, each told the secret to a further three friends. And so it continued, each quarter of an hour. How many people knew the well kept secret at the n^{th} hour?

With regard to types of relevant and pertinent mathematical competence in the field of education, they are defined as Competence Type 1: *Knowledge and Development of*

Mathematical Procedures, which includes proposing, formulating and resolving routine types of problem in a real, realist, fantasy and purely mathematical context, that require the establishment of connections, the extension of the mathematical concepts and the use of mathematical argument for their solution. Basically, it consists in problems with the most common types of calculus and definitions that appear in conventional mathematical evaluations.

Example: 10.4% of intoxications are provoked by domestic pesticides**,** explain.

Competence Type 2 – *Problem Solving*: includes formulating and solving types of routine problems in a real, realistic, fantasy and purely mathematical context, that require the establishment of connections for their solution.

Example: During an experiment, it is observed that when a rubber ball falls, it bounces vertically and on each bounce the ball rises to one third of the height reached on the previous bounce. If the ball were to fall initially from a height of 6 metres, how many meters would it cover if it were to bounce four times?

Competence Type 3 – *Formulation of and Solving Non-Routine Problems:* includes the decoding of different ways of presenting mathematical situations, translating natural language to symbolic/formal language, that is, mathematical knowledge that involves the capacity to generalise.

Example: A culture contains 100 cells per square millimetre, supposing the population duplicates per generation, how many cells would there be after n + 1 generations?

Result

The highest achievement percentages were recorded in the problems corresponding to purely mathematical context problems - competence type 2, realist context - competence type 2 and fantasy context problem -competence type 3.

The lowest achievement percentages were recorded in the resolution of the following problems and context types: Routine problem of fantasy context and Competence Type 2-Solving of Routine Problems; Routine problem of Realistic context and Competence Type 2-Solving of Routine Problems; Non routine problem of context and Competence Type 3 Formulation of and Solving Non-Routine Problems.

Problem solving and pertinent skills at the different education levels are considered to be key factors in mathematics education in Iberoamerica and are associated with many of the central thematic elements proposed in curricula today.

References

Beaton, A., Mullis, I.V.S., Martin, M.O., González, E.J., Kelly, D.L. y Smith, T.A TIMSS Assessment Frameworks and Specifications 2003. *TIMSS International Study Center 2003*

Chamorro, M. (2003). Las dificultades de lectura y comprensión de los problemas matemáticos escolares. *UNO: Revista de Didáctica de las Matemáticas*, 33, 99-119.

Díaz. V., Poblete A. (1994). Evaluación de los aprendizajes matemáticos en la enseñanza secundaria en el marco de la reforma educacional . *Proyecto de la Comisión Nacional de Investigación Científica y Tecnológica CONICYT.* Fondecyt 1990558

Díaz. V., Poblete A. (2002). La habilidad de resolución de tipos de problemas en matemática y sus logros: ¿Un desafío en la reforma educacional en Chile? *Revista Boletín de Investigación Educacional*, 17, 204-219.

Díaz. V., Poblete A. (2004). Evaluación longitudinal de aprendizajes matemáticos, objetivos transversales e indicadores de contexto. *Proyecto de la Comisión Nacional de Investigación Científica y Tecnológica CONICYT*. Fondecyt 1040035.

Gil, G.,(2002) El proyecto internacional para la producción de indicadores de resultados Educativos de los alumnos. *Proyecto PISA de la OCDE*.

Poblete A. Díaz V. (2003). Evaluación de las competencias profesionales del profesor de matemáticas en el marco de la reforma educacional. *Proyecto de la Comisión Nacional de Investigación Científica y Tecnológica CONICYT*. Fondecyt 1010980.