Mathematics Education: Pastoral Bildung - Or Anti-Pastoral Enlightenment

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Applying a postmodern philosophical perspective to mathematics education reveals different kinds of mathematics and different kinds of education and different kinds of philosophy. Based upon the ancient Greek controversy between the sophists and philosophers as to the nature of knowledge, two different forms of schooling has developed, an enlightenment school abstracting categories from physical examples; and a pastoral school exemplifying metaphysical categories; as well as two different kinds of mathematics, enlightenment mathematics seeing the world as the roots of mathematics, and pastoral mathematics seeing the world as applying mathematics.

Pre-modern and Modern Mathematics

Pre-modern Enlightenment mathematics presented mathematics as a natural science. Exploring the natural fact multiplicity, it established its definitions as abstractions from examples, and validated its statements by testing deductions on examples. Inspired by the invention of the set-concept, modern mathematics turned Enlightenment mathematics upside down to become 'metamatics' that by defining its concepts as examples of abstractions, and proving its statements as deductions from meta-physical axioms, needs no outside world and becomes entirely self-referring.

However, a self-referring mathematics soon turned out to be an impossible dream. With his paradox on the set of sets not being a member of itself, Russell proved that using sets implies self-reference and self-contradiction known from the classical liar-paradox 'this statement is false' being false when true and true when false: 'Definition: $M = \{ A \mid A \notin A) \}$. Statement: $M \in M \Leftrightarrow M \notin M$'.

Likewise, without using self-reference it is impossible to prove that a proof is a proof; a proof must be defined. And Gödel showed that theories can't be proven consistent since they will always contain statements that can neither be proved nor disproved.

Being still without an alternative, the failing modern mathematics creates big problems to mathematics education as e.g. the worldwide enrolment problems in mathematical based educations and teacher education (Jensen et al, 1998).

Pastoral and Anti-pastoral Philosophy

Ancient Greece saw a fierce controversy between two different forms of knowledge represented by the sophists and the philosophers. The sophists warned that to protect democracy, people needed to be enlightened to tell choice from nature in order to prevent patronization presenting its choices as nature. The philosophers argued that patronization is the natural order since everything physical is an example of meta-physical forms only visible to the philosophers educated at Plato's academy, who then should become the natural patronising rulers.

The Greek democracy vanished with the Greek silver bringing wealth by financing trade with Far-East luxury goods; later the trade was reopened by German silver financing the Italian Renaissance; and by silver found in America. Robbing slow

Spanish silver ships returning over the Atlantic was no problem to the English; finding a route to India on open sea was. Until Newton found that when the moon falls to the earth as does the apple, it is not obeying the unpredictable will of a meta-physical patronizor only attainable through faith, praying and church attendance; instead it obeys its own predictable physical will attainable through knowledge, calculations and school attendance.

This insight created the Enlightenment period: when an apple obeys its own will, people could do the same and replace patronization with democracy. Two were installed, one in US, and one in France. US still has its first republic, France now has its fifth. The German autocracy tried to stop the French democracy by sending in an army. However, the German army of mercenaries was no match to the French army of conscripts only to aware of the feudal alternative to stopping the German army. So the French army stopped the German army and later occupied Germany.

Unable to use the army, the German autocracy instead used the school to stop enlightenment spreading from France. Humboldt was asked to create an elite school. Using Bildung as counter-enlightenment, he created a school-system leading to the Humboldt University, which uses Luhmann System Theory to defend its chosen self-reference as nature (Luhmann 1995).

Inside the EU the sophist warning is kept alive only in France in the postmodern thinking of Derrida, Lyotard and Foucault warning against pastoral patronising categories, discourses and institutions presenting their choices as nature (Tarp 2004). Derrida recommends that pastoral categories be 'deconstructed'. Lyotard recommends the use of postmodern 'paralogy' research to invent alternatives to pastoral discourses. And Foucault uses the term 'pastoral power' to warn against institutions legitimising their patronization with reference to categories and discourses basing their correctness upon choices claimed to be nature.

In descriptions, numbers and words are different as shown by the 'number & word dilemma': Placed between a ruler and a dictionary, a'17 cm long stick' can point to '15', but not to 'pencil', thus being able itself to falsify its number but not its word, which makes numbers nature and words choices, becoming pastoral if suppressing their alternatives; meaning that a thing behind a word only shows part of its nature through a word, needing deconstruction to show other parts.

Thus anti-pastoral sophist research doesn't refer to but deconstruct existing research by asking 'In this case, what is nature and what is pastoral choice presented as nature?' To make categories, discourses and institutions anti-pastoral they are grounded in nature using Grounded Theory (Glaser et al 1967), the method of natural research developed in the other Enlightenment democracy, the American; and resonating with Piaget's principles of natural learning (Piaget 1970) and with the Enlightenment principles for research: observe, abstract and test predictions.

The Nature of Numbers

Feeling the pulse of the heart on the throat shows that repetition in time is a natural fact; and adding one stick and one stroke per repetition creates physical and written multiplicity in space.

A collection or total of e.g. eight sticks can be treated in different ways. The sticks can be rearranged to an eight-icon 8 containing the eight sticks, written as 8. The sticks can be collected to one eight-bundle, written as 1 8s. The sticks can be 'decimal-counted' in 5s by bundling & stacking, bundling the sticks in 5s and stacking the 5-bundles in a left bundle-cup and stacking the unbundled singles in a right single-cup. When writing down the counting-result, cup-writing gradually leads to decimal-writing where the decimal separates the bundle-number from the single-number: 8 = 1 5s + 3 1s = 1)3) = 1.3 5s = 13 if leaving out the decimal and the unit.

So the nature of numbers is that any total can be decimal-counted by bundling & stacking and written as a decimal number including its unit, the chosen bundle-size. Choosing ten means that no icon for ten is needed since the bundle is 1.0 bundle. Choosing eight instead, ten becomes 12, and 10 becomes eight.

The Nature of Operations

Operations are icons describing the process of counting by bundling & stacking. The division-icon '/2' means 'take away 2s' when counting in 2s, 8/2 = 4. The multiplication-icon '4*' means 'stacked 4 times' when stacking 2-bundles, T = 4*2. Subtraction '- 2' means 'take away 2' when taking away the bundles to see what rests as unbundled singles, R = 9 – 4*2. And addition '+2' means 'plus 2' when adding 2 singles to the stack of bundles as a new stack of 1s making the original stack a stock of e.g. T = 2*5 + 3*1, alternatively written as T = 2.3 5s if using decimal-counting.

Thus the full process of 're-counting' or 're-bundling' 8 1s in 5s can be described by a 'recount or rebundle formula' T = (T/b)*b saying the total T is first counted in bs, then stacked in bs, together with a 'rest formula' finding the rest:

T = (8/5)*5 = 1*5 + 3*1 = 1.3*5 since the rest is R = 8 – 1*5 = 3.

The Nature of Formulas

Using these formulas, the counting result can be predicted on a calculator thus becoming a number-predictor. This shows the strength of mathematics as a language for number-prediction able to predict mentally a number that later is verified physically in the 'laboratory'. Historically, this enabled mathematics to replace pastoral belief with prediction, and to become the language of the natural sciences.

The Nature of Equations

The statement 2*4 + 1 = 9 describes a bundling where 2 4-bundle and 1 single is re-bundled to 9 1s. The equation x*4 + 1 = 9 describes the reversed bundling asking how many 4-bundles that together with 1 single can be re-bundled to 9 1s. Obviously, we must take the 1 single away from the 9 1s and count the rest in 4s. So technically, moving numbers to the other side changing their calculation sign solves an equation: If x*4 + 1 = 9, then x*4 = 9-1 = 8, and x = 8/2 = 2.

Enlightenment Mathematics and Pastoral Mathematics

In primary school, an enlightenment curriculum will focus on the nature of numbers, operations and calculations to be learned through counting by bundling & stacking reported by cup-writing and decimals, 8 = 1 5s + 3 1s = 1)3) = 1.3 5s, in accordance with the Piaget 'from hand to head' principle of natural learning; and postponing the

introduction of ten and addition until after several examples of the fact that for any bundle-size, its icon will not be used since a full bundle will always be counted as 1.0 bundles or plain 10 if leaving out both the decimal and the bundle-size.

The pastoral curriculum introduces the 'natural' numbers one by one using the follower-principle. This leads to introducing 10 as the follower of 9 and in this way quickly introducing 2digit numbers and place values. Later comes multi-digit numbers. Likewise addition is introduced first to practise earlier numbers adding up to the actual number. Then subtraction is introduces as taking away and counting up to. Multiplication and the tables follow; and in the end division and simple fractions.

In middle school, an enlightenment curriculum will focus on the nature of per-numbers and triangles. Per-numbers occur when double-counting a quantity in two different units leads to fractions and percentages, 2$ per 3m = 2$/3m = 2/3 $/m. Recounting now is called proportionality. When adding fractions and percentages the units are includes as in integration. Formulas with two unknowns are graphed. Formulas with one unknown are equations solved using reversed calculations first reducing a multiple calculation to a single by placing the hidden parentheses, and then moving numbers to the other side reversing their calculation signs. Geometry is earth-splitting using triangles inside or outside coordinate systems, seeing a right-angled triangle as a rectangle halved by a diagonal, where the height and length can be recounted in diagonals making sine, cosine and tangent percentages.

The pastoral curriculum enlarges the number domain with fractions, and with decimals and percentages both defined as examples of fractions. Again the order of operations is maintained starting with addition of fractions including factorisation of 'natural' numbers in prime numbers. Equations are introduced and solved by the neutralising method. Proportionality is introduced as an example of equations and of a function that is graphed. Algebraic expressions are introduced in equations to be factorised and simplified, and to be added as algebraic letter fractions. In geometry the focus is on 2- and 3-dimensional forms and translation groups.

In high school, an enlightenment curriculum will focus on adding per-numbers, where adding constant per-numbers leads to power; and adding variable per-numbers leads to integration where primary school's adding stacks in combined bundle-sizes, and middle school's adding fractions with units are generalising to finding the total as the area under a per-number graph. Reversed addition then leads to roots and logarithms, and to differentiation finding the per-number as the gradient on a total-graph: 2s at 3m/s + 5s at ?m/s = 7m at 4m/s, ? = (7*4-2*3)/5 = T/ x.

The pastoral curriculum enlarges the number domain and operations with irrational and real numbers, and with power, root and log. The function concept is claimed to be the foundation of high school mathematics; and is defined as an example of a relation between two sets. Linear and exponential change is presented as examples of functions. The quadratic function is given an extended treatment. Its graph is studied using translations, and its formula is thoroughly factorised. Calculus is introduced as an example of the concept limit used to exemplify the concept continuity and differentiability and to define the gradient by the first principle and the integral as a

Riemann sum. Geometry introduces coordinate geometry and vector geometry presenting a vector as an equivalence set of parallel arrows with the same length.

Enlightenment Schools and Pastoral Schools

After only a week, an ethnographer will see a fundamental difference between universities in North America and in the EU. At the first place the students are aged 19, at the second 23. At the first place the students have chosen their own combination of modules accessible for all; at the second place they are forced to follow one of several pre-designed educations only accessible to those with the highest marks. At the first place the students already took some university modules at the last year in high school; at the second place this is not possible. At the first place high school is attended by all and most go on to university where around the half gets a bachelor degree; at the second place only the best half of a year group is allowed to enter high school, and only best half is allowed to go on to university where only the half graduates after having been forced to include a university directed master degree in their exam. At the first place some students are supplementing their bachelor degree with new modules in order to change career e.g. from teaching to engineering; at the second place they have to start all over. At the first place parents have different careers in their lifetime; at the second place parents are bound to the office they are educated for. At the first place some students are studying education; at the second place education has to be studied outside the university. At the first the bachelor-degree takes four years and can be combined from different universities; at the second place the bachelor-degree takes three years and must be finished at one university, so the compulsory master-degree can't be taken outside the EU.

Looking in the literature for explanations for this difference soon leads to Humboldt:

> Our universities have a monastic origin, and they have specialized in being centers of higher learning, functions originally given by the Church to monasteries. (..) The form of the university most familiar to us today is mainly a Prussian invention whose architect and champion was Wilhelm von Humboldt (..) The collegial system and its related peer review structures centered on an effort to gain intellectual freedom from the constraints of theological doctrine and political manipulation. Although addressing this problem was obviously important, the solution adopted has subsequently done much to weaken the social articulation of the university to all groups other than powerful elites. (..) Not surprisingly, society at large occasionally thinks it should be getting a more useful return for its investment and the freedom it gives to the professoriate. This situation is predictable because the autopoetic research process provides important supports for intellectual freedom but simultaneously opens the door to useless research and academic careerism divorced from attention to important public social issues (Greenwood et al in Denzin et al. 2000: 85-89)

Facing a Choice: Democratic Enlightenment or Pastoral Patronisation

The ancient Greek controversy on the nature of knowledge between the sophists warning against patronisation and the philosophers recommending patronisation has been running up through human history. The Christian church gladly accepted the idea of metaphysical patronisation and transformed the Plato academy into a monastery. Brahe, Kepler and Newton rebelled against the library's monopoly on knowledge by pointing to laboratory observations as the knowledge source. This created the Enlightenment period believing that when enlightened through schooling, people could replace patronisation with democracy.

However, the two democracies installed, the American and the French, developed different forms of anti-pastoral thinking. The French post-structuralism is described above. America developed pragmatism created by Peirce and James arguing that the focus should be shifted away from laws to habits and ability to work. Later American pragmatism developed to Blumer's 'symbolic interactionism' developing its own methodology called 'Grounded Theory' grounding its categories and relations in data and being sceptical towards existing research categories (Tarp 2004).

To prevent enlightenment and democracy to spread from France, Germany invented Humboldt Bildung and Humboldt universities refusing to receive students without an entrance exam from a Humboldt-gymnasium only allowing the best half to enter, and the best half to go on, thus effectively identifying the elite. Today EU still has Humboldt universities while the rest of the world has enlightenment universities.

The invention of the controversial set-concept allowed mathematics to become self-referring defining its concepts as examples of sets. This 'metamatics' defining concepts as examples of abstractions instead of as abstractions from examples, came to mathematics education as modern mathematics, gladly accepted and guarded at Humboldt Bildung schools, and reluctantly adjusted at enlightenment schools.

However, a grounded approach to mathematics education reveals the existence of 'mathematism' (Tarp 2004) being true in the library, but not in the laboratory where e.g. 2+3 = 5 has countless counter-examples: 2weeks+3 days = 17 days, 2m+3cm = 203 cm etc.; in contrast to the statement that 2*3 = 6 stating that 2 3s can be recounted as 6 1s. Mixing metamatics and mathematism to 'metamatism' makes mathematics pastoral by suppressing its natural alternative, mathematics as a natural science studying multiplicity by counting and adding.

Questions

Should teachers be enlightened on the difference between enlightenment and Bildung schools? And on the different attitudes towards patronization, warned against by the sophists and recommended by the philosophers, as expressed today in modern and postmodern philosophy? And on the difference between mathematics, metamatics and mathematism? And on the existence of 1digit mathematics (Zybartas et al 2005)? And that mathematics becomes pastoral by suppressing its alternative, enlightenment mathematics? Or should teachers just unenlightened follow orders?

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